Oscillons in Scalar Field Theories: Applications in Higher Dimensions and Inflation

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Abstract

The basic properties of oscillons – localized, long-lived, time-dependent scalar field configurations – are briefly reviewed, including recent results demonstrating how their existence depends on the dimensionality of space-time. Their role on the dynamics of phase transitions is discussed, and it is shown that oscillons may greatly accelerate the decay of metastable vacuum states. This mechanism for vacuum decay – resonant nucleation – is then applied to cosmological inflation. A new inflationary model is proposed which terminates with fast bubble nucleation.

1 Introduction

During the past three decades, static, spatially-localized field configurations have been of great interest in relativistic field theories [1, 2]. These nonperturbative solutions of nonlinear field equations have been shown to exist in a wide variety of models, with or without nontrivial vacuum topology. Configurations with stability guaranteed by vacuum topology are called topological solitons. Examples include kinks, strings, monopoles, and textures [3]. When the configuration's stability comes from conserved charges, they are called Q-balls [4] or nontopological solitons [2]. Both topological and nontopological solitons have many applications in high-energy particle physics and cosmology [1, 3]. Their widespread appeal relies on two key properties: first, they are stable, that is, they maintain their spatial profile through time. (Of course, if the configurations are allowed to scatter they may or not remain stable and form bound states.) Second, their energy density is spatially localized. Thus, they have been used to model hadrons [1, 2], as possible signatures of primordial phase transitions, or to generate the large-scale structure of the Universe [3].

The purpose of the present work is to present results pertaining to a related but, in this author's view, equally important class of spatially-localized field configurations. Their key difference from the solitons mentioned above concerns their temporal behavior: they are *time-dependent*, as opposed to static, solutions of the equations of motion. What makes them potentially interesting for many applications is that, even though they will eventually radiate their energy to spatial infinity, the process is quite slow. If their lifetime is longer than the typical time-scale in the system they will, for all practical purposes, behave as solitons. The remarkable fact about these long-lived, time-dependent configurations – called oscillons – is that they owe their longevity to the nonlinearities in the system: no globally-conserved charges or topologically nontrivial

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boundary conditions are needed (although they may help extend the oscillon's lifetime). As such, they can be found in a much broader class of models.

In the next section oscillons are briefly introduced, followed by an exploration of their properties in an arbitrary number of spatial dimensions [5]. In section 3, the possible effect of oscillons on first order phase transitions is discussed [6]. Finally, in section 4, they are applied to cosmology, in particular to a new two-field model of inflation that ends with rapid bubble nucleation.

2 Oscillons in d Dimensions: Basics

Oscillons were first shown to exist in the context of simple scalar field theories with symmetric and asymmetric double-well potentials [7, 8]. They were found to be spherically-symmetric, time-dependent solutions of the equations of motion obtained from the d-dimensional Lagrangian[5],

$$L = c_d \int r^{(d-1)} dr \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2 - V(\phi) \right) \ , \tag{1}$$

where a dot means time derivative. The d-dimensional spatial volume element can be written as $d^d x = c_d r^{(d-1)} dr$, where $c_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a d-dimensional sphere of unit radius. An oscillon is a time-dependent, spatially-localized solution of the equation of motion $\ddot{\phi} - \nabla_d^2 \phi = -V'$, characterized by a nearly constant energy, $E_{\rm osc}$, and by a persistent nonlinear oscillation about its core at $r = 0$. Think of a rubber sheet that is pinched and then let go. An oscillon would be a localized oscillation on the sheet that doesn't get radiated away for a long time. To find an oscillon, write the scalar field as

$$\phi(t, r) = [\phi_c(t) - \phi_v] \exp[-r^2/R^2] + \phi_v \ , \tag{2}$$

where $\phi_c(t)$ is the core value of the field $[\phi(t, r = 0)]$, and ϕ_v is its asymptotic value at spatial infinity, determined by $V(\phi)$. Using the Gaussian profile of eq. 2, it has been shown that oscillons are present whenever the energy of the initial configuration $\phi(0, r)$ is larger than $E_{\rm osc}$ and $R \geq R_{\rm osc}$, where $R_{\rm osc}$ is the oscillon radius that can be obtained analytically [7]. In addition, the potential must have a portion where $V'' < 0$ and the field must probe this portion at $r \sim 0$. In fact, the oscillon owes its stability to this "spinodal instability" in the potential [7].

Writing $A(t) = \phi_c(t) - \phi_v$ and parameterizing the potential as [5]

$$V(\phi) = \sum_{j=1}^{h} \frac{g_j}{j!} \phi^j - V(\phi_v) \ , \tag{3}$$

where the g_j's are constants and the vacuum energy $V(\phi_v)$ is subtracted from the potential to avoid spurious divergences upon spatial integration, one obtains

the equation of motion

$$\ddot{A} = -\frac{d}{R^2}A - \sum_{n=2}^{h}\left(\frac{2}{n}\right)^{d/2}\frac{1}{(n-1)!}V^n(\phi_v)A^{n-1} \ . \tag{4}$$

Expanding the amplitude as $A(t) = A_0(t) + \delta A(t)$ and linearizing the eom with $\delta A \sim e^{i\omega t}$, it is found that oscillons may only exist if $\omega^2 < 0$. For quartic potentials with $g_4 > 0$ (that is, symmetric and asymmetric double wells), this conditions implies

$$R^2 \geq \frac{d}{\left[\frac{1}{2}\left(\frac{2^{3/2}}{3}\right)^d \frac{(V''')^2}{V^{IV}} - V''\right]} \ . \tag{5}$$

That is, oscillons only exist for lumps above a certain critical size. For $d = 3$, $R_{\rm osc} \simeq 2.42$, while for $d = 6$, $R_{\rm osc} \simeq 7.5$. Note that since the denominator of eq. 5 must be positive, there is also an upper critical dimension for oscillons to exist,

$$d \leq {\rm Int}\left[\frac{\ln 2\frac{V''V^{IV}}{(V''')^2}}{\ln\left(\frac{2^{3/2}}{3}\right)}\right] \ . \tag{6}$$

For a symmetric double-well, $d_c = 6$ [5].

The fact that for these simple models oscillons can only exist below a certain number of spatial dimensions raises an interesting possibility. Let's assume that this model carries the information of the scalar sector of a more realistic field theory. (Farhi *et al.* recently found oscillons in a $SU(2)$ model [9].] Then, if extra dimensions exist and are large [10], it is conceivable that oscillons could be produced in collisions with high enough energy, $E_{\rm col} > E_{\rm osc}$. Since their lifetime is much longer than the typical time-scales in the system, they would appear as a late fireball concentrated in a volume $\sim R^d_{\rm osc}$. Furthermore, since $E_{\rm osc} \sim \frac{1}{2}(\pi/2)^{d/2}d^{d-1}$, finding an oscillon would provide information about the dimensionality of spacetime [5]. In other words, oscillons could be used as probes for the existence of higher dimensions.

3 Resonant Nucleation

Recently, oscillons have been shown to emerge dynamically when a system is quenched from a single to a double well potential [11]. This means that, under certain conditions, they are spontaneously produced during the nonequilibrium dynamics of the system. [The same results would be obtained at $T = 0$, as long as the field starts in, say, a Gaussian superposition of momentum modes.] The system was initially prepared in a thermal state on a single well and then quenched to a symmetric double well, as is common in Ginzbug-Landau models of phase transitions [12]. The key point is that the system was prepared so that after the quench the field was still localized in one of the wells. For example, the potential could have been changed from $V(\phi) = (\phi + 1)^2$ to $V(\phi) = \frac{1}{4}(\phi^2 - 1)^2$.

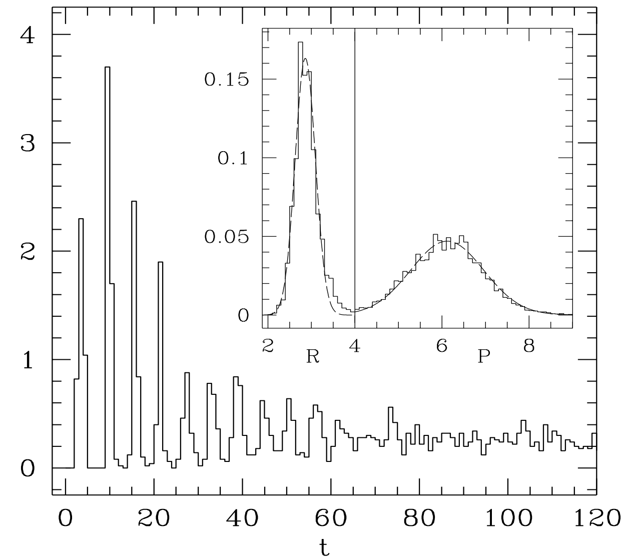

Figure 1: The number of oscillons nucleated between t and $t + \delta t$ at $T = 0.2$, with $\delta t = 1$. The global emergence is evident early in the simulations. Inset: the probability distribution of radii and periods of oscillation for the oscillons nucleated.

The field would then stay trapped around the $\phi = -1$ minimum. This is what happens, for example, during the initial stages of a first order phase transition, when the field is initially in a metastable state. More generally, one can write a potential

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{\alpha}{3}\phi^3 + \frac{\lambda}{8}\phi^4 \ , \tag{7}$$

where α is a tunable interaction. Initially, $\alpha = 0$ and the field is in a single well. Then α acquires a positive nonzero value and the potential becomes a double well. For $\alpha = 3/2$ the double well is symmetric; for $\alpha > 3/2$ the minimum at $\phi = 0$ is metastable [11].

The net effect of the quenching is to decrease the effective mass of the field around the minimum. This key point may not be apparent at tree level, especially around $\phi = 0$. However, one must remember that the mass must be corrected due to thermal and/or quantum fluctuations. One way of incorporating these changes is to adopt a simple Hartree approximation, shifting the mass from m^2 to $m_H^2 = m^2 + \frac{3}{2}\langle\phi^2\rangle$. Before the quench, the Hartree potential is (upon rescaling the couplings) $V_H(\phi) = \frac{1}{2}[1 + \frac{3}{2}\langle\phi^2\rangle]]\phi^2$. After the quench,

$$V_H(\phi) = \left[1 - m_H^2\right]\phi + \frac{1}{2}m_H^2\phi^2 - \frac{\alpha}{3}\phi^3 + \frac{1}{8}\phi^4 \ . \tag{8}$$

Thus, the quench induces a shift in the minimum from $\phi = 0$ to $\phi \simeq \frac{3}{2}\langle\phi^2\rangle$.

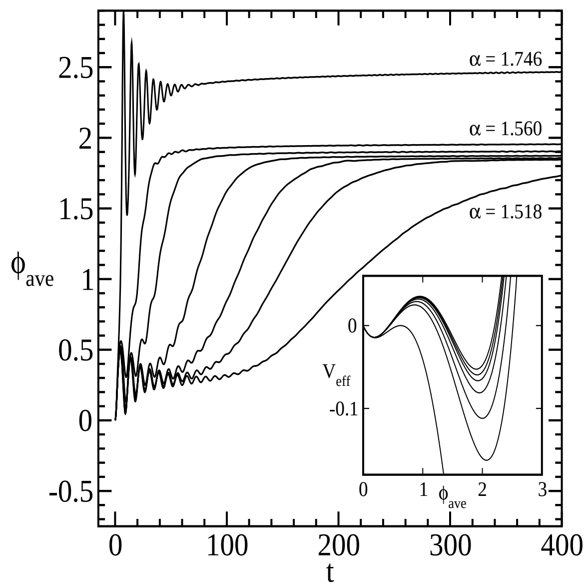

Figure 2: The evolution of the order parameter $\phi_{\rm ave}(t)$ at $T = 0.22$ for several values of the asymmetry in $d = 2$. (From left to right, $\alpha = 1.746, 1.56, 1.542, 1.53, 1.524, 1.521, 1.518$. Each curve is an ensemble average over 100 runs. The inset shows the effective Hartree potential for the same values of α.

This shift, plus the fact that $V_H(\phi)$ acquires a negative derivative at the origin, explains what happens: the field's zero mode starts oscillating about the shifted minimum, with an amplitude controlled both by α and T. These oscillations may induce parametric amplification of a band of k-modes which, for a range of parameters, trigger the emergence of oscillons [11]. The oscillations transfer their energy to higher momentum modes via nonlinear scattering and the oscillons eventually disappear as the system reaches equipartition.

Fig. 1 shows the number of oscillons nucleated as a function of time. Two points are important: first, oscillons emerge in synchrony. Second, this initial synchrony is gradually lost as equipartition is achieved.

What happens if the potential is asymmetric? In this case, one is concerned with the way a quenched field will decay from a metastable (or false vacuum) state to the global minimum. As is well known, if the field is initially well-localized about the false vacuum state, it will decay by nucleating bubbles of a critical size with nucleation rate per unit volume $\Gamma \simeq T^4 \exp[-E(T)/T]$, where T is the temperature and $E(T)$ is the energy of the critical bubble or bounce configuration [13]. At $T = 0$, the quantum decay rate is $\Gamma \simeq M^4 \exp[-S_b]$, where M is the typical mass scale and S_b is the $d+1$-dimensional Euclidean bounce action.

As in the case of a symmetric double well, the fast quench will generate a

distribution of oscillons for a certain range of temperatures. The efficiency of this production mechanism is sensitive to the quenching time scale, $\tau_{\rm quench}$. If $\tau_{\rm quench} \gg \tau_0$, where τ_0 is the equilibration time scale of the field's zero-mode, oscillons will be produced. In Fig. 2 we show the variation of the average value of ϕ after the quench. It can be seen that close to degeneracy ($\alpha \sim 3/2$) the field spends a long time oscillating about the metastable minimum before decaying to the ground state.

The key point is that a fast enough quench will greatly affect the decay rate [6]. As shown in Ref. [6], a fast quench will promote a much faster decay: for a range of parameters, the decay rate goes from an exponential to a power law suppression,

$$\Gamma \simeq T^4[E(T)/T]^{-B} \ , \tag{9}$$

where B can be obtained numerically. For $d = 2$, it was shown that $B \simeq 2.9 \pm 0.46$ for the range of temperatures where oscillons are copiously produced [6]. In Fig. 3 the decay time-scale is plotted against the bounce energy for different temperatures. It is clear that a power law is an excellent approximation.

Work in $d = 3$ is currently under way. Preliminary results indicate that the power law behavior remains, with $B \sim 2$.

The main consequence of the above result is that whenever the quenching happens fast, using an exponentially-suppressed decay rate as dictated by false-vacuum decay theory is simply incorrect. Since $\Gamma \sim \exp[-E(T)/T]$ (or $\sim \exp[-S_b]$ at $T = 0$) is widely used in applications ranging from early universe cosmology (in particular at GUT scales, where H is fast) to models of quark-hadron phase transitions in collisions such as in RHIC, one should be careful about their range of validity. It may very well be that transitions that were considered slow in the past are actually quite fast. As an application of what was called "resonant nucleation" (RN) in Ref. [6], I will briefly investigate its possible effects on inflation.

4 Resonant Inflation: Can Old Inflation be Rescued?

The simple elegance of the original "Old" Inflation (OI) scenario proposed by Guth in 1981 has, since then, inspired many variations [14, 15]. More than just the elegance of its formulation, based on a single scalar field decaying from an initial metastable state to a lower-energy state by bubble nucleation, the original OI model had a clear connection with particle physics: the inflaton was to be the same scalar field promoting the symmetry breaking of Grand Unified models, linking early-Universe cosmology to high-energy particle physics. In fact, it is this particle physics connection that motivated and motivates the widespread use of scalar fields in early-Universe physics.

Unfortunately, Guth's original proposal didn't work. As he himself argued, and then Linde, and Albrecht and Steinhardt [16], the bubble-nucleation rate could not compete with the exponential expansion rate of the Universe: the

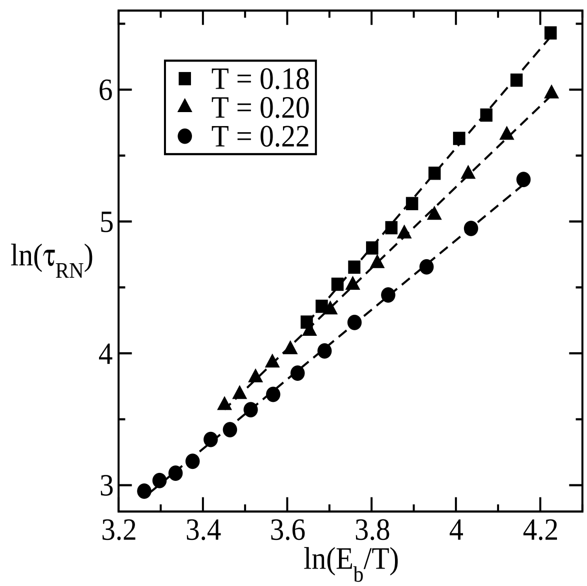

Figure 3: The decay time-scale $\tau_{\rm RN}$ for resonant nucleation as a function of critical nucleation free-energy barrier $E(T)/T$ at $T = 0.18, 0.2$, and $T = 0.22$. The best fit is a power-law with exponent $B \simeq 2.44$ for $T = 0.22$, and $B \simeq 3.36$ for $T = 0.18$.

transition would never end. Roughly, while bubble walls expanded with the speed of light, their centers receded from each other exponentially fast, making it impossible for the walls to touch, the bubbles to coalesce, and the transition to complete. Old Inflation gave rise to a universe with inhomogeneities incompatible with the observed smoothness of the cosmic microwave background [17]. Guth and Weinberg [18], and later Turner, Weinberg, and Widrow [19], performed a detailed analysis of the constraints needed to render OI and OI-inspired scenarios viable. They concluded that a strong (or, equivalently, slow) first order phase transition based on a single scalar field could not be made to work: the ratio of decay rate to the expansion rate per unit volume $[H^4 \simeq (T^2/M_{\rm Pl})^4]$, had to be sufficiently small

$$\varepsilon \equiv \Gamma/H^4 \leq 10^{-4} \ , \tag{10}$$

initially, so that early bubbles didn't produce inhomogeneities during nucleosynthesis and on the CMB. (For $\Gamma \simeq T^4 \exp[-E(T)/T]$ and $T_{\rm GUT} = 10^{15}$ GeV, this implies that $E(T)/T \geq 46.1$ initially). On the other hand, it had also to grow by the end of inflation ($\varepsilon \rightarrow 9\pi/4$) to guarantee that the transition was completed [19]. [This implies $E(T)/T \leq 34.9$.] In other words, successful inflation forced the decay rate to be time-dependent: small at the beginning of inflation and of order unity at the end. As further work has shown, this could be achieved by invoking more fields [20] and/or a nonminimal gravitational coupling [21].

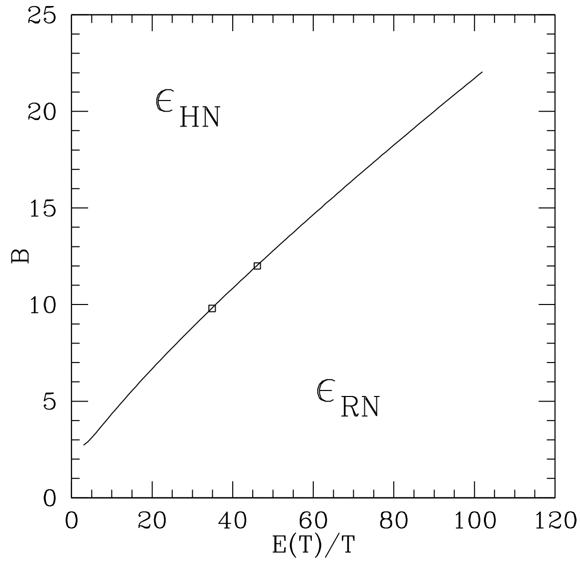

Figure 4: Comparison of homogeneous nucleation (HN) and resonant nucleation (RN) with power B in an expanding Universe. For a fixed B and nucleation barrier $\beta = E(T)/T$ (or S_b at $T = 0$), the line denotes equality. Values above the curve imply faster HN, while those below imply faster RN.

Given what we have learned in the previous section about resonant nucleation, it is natural to wonder whether such effects can play a role on inflation. If we write $\varepsilon_{HN} \simeq T^4 \exp[-E(T)/T]$ to represent the ratio of eq. 10 using the homogeneous nucleation rate, and $\varepsilon_{RN} \simeq T^4[E(T)/T]^{-B}$ the ratio using the RN rate, equality is attained whenever

$$B = \beta/\ln\beta \ , \tag{11}$$

where $\beta \equiv E(T)/T$ (or $\equiv S_b$ at $T = 0$). In Fig. 4 B is shown for representative values of the nucleation barrier β. The line denotes $\varepsilon_{\rm HN}/\varepsilon_{\rm RN} = 1$. The squares denote the limits imposed by the inflationary constraints of Ref. [19]. For these values of β, unless $B \geq 9$, which is very unlikely, resonant nucleation rates are always faster. In $d = 2$, where $B \simeq 3$, it is clear that $\varepsilon_{HN}/\varepsilon_{RN} < 1$ for all realistic values of β, not a surprising result.

Why is this useful for inflation? For successful inflation with HN, the constraints of Ref. [19] limit the nucleation barrier β to be fairly small O(~ 40). [See Fig. 4.] However, calculations of bounce actions show that β usually scales with inverse powers of coupling constants. These two requirements compete with each other, making it hard to have small nucleation barriers with small couplings. Applying the percolation constraint to the RN rate, one obtains $\beta^B \sim 10^{16}$. For $B = 3$, this gives $\beta \sim 10^{16/3}$: small couplings (or, equivalently, large barriers) are easier to accommodate with RN, the first reason why it may

be useful for inflation.

The second and most important reason is that RN makes it much easier to complete the transition. Clearly, if some mechanism capable of producing the same net effect as the fast quenching responsible for RN was present in the early Universe, an initially slow first order transition could become fast at some point, going from an exponential to a power law decay. In this way, even a potential with a large initial barrier would not be an impediment to the successful termination of inflation. One possible way of implementing RN in cosmology is to invoke a second field ψ that couples to the nucleating field ϕ in a way somewhat reminiscent of hybrid inflation. In that model the inflaton ϕ is coupled quadratically to another scalar field ψ which has a *symmetric* double well potential [20]:

$$V(\phi,\psi) = \frac{1}{4\lambda}\left(M^2 - \lambda\psi^2\right)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\psi^2 \ . \tag{12}$$

Inflation is driven by the energy density $V(\phi, 0)$ while the inflaton (ϕ) is rolling down along the $\psi = 0$ valley [15]. As ϕ reaches a critical value, ψ becomes spinodally unstable and quickly rolls to one of the minima (or both, but this creates domain walls, another problem), terminating inflation abruptly.

The key difference with the mechanism being proposed here is that bubble nucleation still occurs at the end of inflation. A possible name is hence *resonant inflation* (RI): it blends OI with the physics of resonant nucleation.

Modify the potential for the field ϕ that gives rise to RN (e.g. eq. 7) by coupling another field (ψ) quadratically to it as follows,

$$V(\phi,\psi) = \frac{1}{2}\left(m^2 + g^2\psi^2\right)\phi^2 - \frac{\alpha}{3}\phi^3 + \frac{\lambda}{8}\phi^4 + \frac{1}{2}m_\psi^2\psi^2 + |V(\phi_+, 0)| \ , \tag{13}$$

where ϕ_+ is the value of ϕ at the global minimum of $V(\phi, 0)$ so that $V(0,\psi)$ provides the net vacuum energy responsible for inflation. [Note that here the inflaton is ψ.) Inflation lasts while ψ is rolling down the $\phi = 0$ valley. Notice that the mass term for ϕ, $M_\phi^2 = m^2 + g^2\psi^2$, decreases as ψ rolls down its potential. While $M_\phi^2 > \alpha^2/2\lambda$, the only minimum in the ϕ direction is at $\phi = 0$. However, as ψ decreases, M_ϕ^2 will eventually drop below $\alpha^2/2\lambda$ and a new minimum will appear at $\phi_+ = \frac{\alpha}{\lambda}\left[1 + (1 - 2M_\phi^2\lambda/\alpha^2)^{1/2}\right]$. At $M_\phi^2 = \frac{4\alpha^2}{9\lambda}$, the two minima are degenerate. At this point, as ψ continues to approach zero, the minimum at $\phi = 0$ becomes metastable. Oscillations in ϕ, induced by the decrease in its mass, will induce RN. This will be true as long as the decrease in M_ϕ, $\dot{M}_\phi \simeq \frac{g^2}{m}\psi\dot{\psi}$, is fast enough. [It was assumed for simplicity that $g^2\psi^2/m^2 \ll 1$ which is not true for very small ψ.]

For RI to work, $\dot{M}_\phi/M_\phi < H$ during inflation and $\dot{M}_\phi/M_\phi > H$ after it. During inflation, with a slow-roll approximation, $\psi\dot{\psi} \simeq -\frac{m_\psi^2}{3H}\psi^2$. We then obtain,

$$\dot{M}_\phi \simeq -g^2\frac{m_\psi^2}{3Hm}\psi^2 \ . \tag{14}$$

Also, if N is the number of e-folds, $\psi_e^2 = \psi_i^2 - \frac{M_{Pl}^2}{2\pi}N$, where $\psi_{i(e)}$ is the value of the field ψ at the beginning (end) of the inflationary period. [For simplicity, it was assumed that during inflation $\frac{1}{2}m_\psi^2\psi^2 > |V(\phi_+, 0)|$, that is, inflation is dominated initially by the vacuum energy of the inflaton field ψ.] Slow-roll ends when $\psi_e^2 \leq M_{Pl}^2/12\pi$. Using this result and eq. 14, the slow variation of M_ϕ implies, $g^2(12\pi)^{1/2} < (m/M_{Pl})^2$. [For $m \sim 10^{16}$GeV, $g < 4 \times 10^{-4}$.] This condition is also consistent with the approximation $g^2\psi^2/m^2 \ll 1$ for $\psi_i > \psi > \psi_e$, that is, during inflation.

If slow-roll ends when the minimum in ϕ_+ appears, we obtain (this is similar to the critical condition in hybrid inflation [20]),

$$\frac{\alpha_0^2}{2\lambda} = 1 + \frac{g^2 M_{Pl}^2}{12\pi m^2} \ , \tag{15}$$

where we defined for convenience $\alpha \equiv m\alpha_0$. The condition for slow variation of M_ϕ during inflation forces the second term on the rhs of eq. 15 to be very small. Thus, if we want to impose that the ϕ_+ minimum appears close to the end of inflation, we must have $\alpha_0^2/2\lambda \sim 1$, not a difficult condition to satisfy.

As inflation ends, ψ will start rolling down fast towards the $\psi = 0$ minimum and oscillate around it. Since in this regime, $\dot{M}_\phi/M_\phi \sim (g^2/m^2)\dot{\psi}\psi$, the rapid motion of ψ will induce the time-dependence in M_ϕ needed to trigger resonant bubble nucleation. In order for the transition to end successfully, the percolation constraint $\varepsilon_{\rm RN} > 9/4\pi$, must be satisfied. This implies,

$$(S_b)^B < \frac{4\pi}{9}\left(\frac{M_{Pl}}{m}\right)^4 \ . \tag{16}$$

If $m \sim 10^{16}$GeV and $B \sim 2$ (as indicated by preliminary results in $d = 3$), RI terminates efficiently if $S_b \leq 10^6$. Since the inflationary phase is due to the slow-roll dynamics of the ψ field and not by the metastable field ϕ, the percolation constraint can be satisfied by a wide range of couplings. Also, since $\phi = 0$ only becomes metastable *after* the end of slow roll, there is no need to impose the big bubble constraint: once ψ starts rolling fast at the end of inflation, RN will ensue and rapid bubble nucleation and coalescence will quickly reheat the Universe. Although several details remain to be worked out, this preliminary analysis indicates that resonant nucleation can be successfully applied to inflationary cosmology.

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